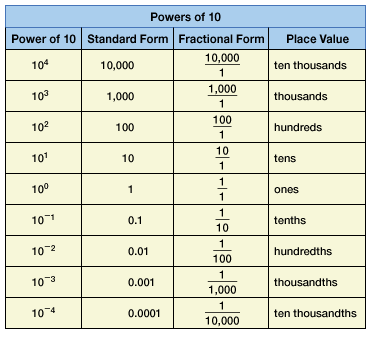
# **Lesson Objectives**

1. Zero and negative exponents
2. Product, Quotient, and Power Properties for Exponents

# **ZERO** and **NEGATIVE** exponents

Zero and negative exponents may be better understood by reviewing place value with base 10:

It turns out that **any** base (except zero)

that has a zero power is equal to 1.

**Property:**

Negative exponents do **NOT** make

negative numbers! They cause a **reciprocal**.

**Property:**

and

“Take the Stairs” for negative exponents.

Cross the line, **change the sign**!

## **Negative Exponents** and **Fractions – “take the stairs” (reciprocal)**

**Property:**  **Property:**

**Simplifying Exponents Tip #1**: Final answer should have **no negative exponents.**

# **Product**, **Quotient**, and **Power** Rules for Exponents

* **Product Rule:**

When multiplying powers w/same base, **ADD** exponents.

* **Quotient Rule:**

When dividing powers w/same base, **SUBTRACT** exponents.

**Simplifying Exponents Tip #2**: Final answer should have **no duplicate variables** – you should see each variable only once.

* **Power Rule:**

When raising a power to a power, **MULTIPLY** exponents.

* **Product to Power Rule:**

The exponent applies to each factor in the parentheses.

* **Quotient to Power Rule:**

The exponent applies to each factor in numerator and denominator.

**Simplifying Exponents Tip #3**: Final answer should have **no parentheses.**

**Simplifying Exponents Tip #4**: A factor with no visible exponent has an **understood** (or implied) **exponent of** **1**. To reduce errors, consider writing in an exponent of 1 in these situations.

**Simplifying Exponents Tip #5**: Simplify exponents that have a constant (number) base. **“Do it!”** For example, don’t leave 25 – change it to 32.

**Other Tips for Success**: Remember to always reduce (simplify) fractions.

Use a calculator for any numerical (non-variable) part.

Watch out with the negatives!

To get better with exponents, you must **PRACTICE!**

* **EXAMPLE:** Evaluate. [R.2.21]

Any base (except zero) raised to the power of 0 equals 1.  **Answer**

* **EXAMPLE:** Simplify the following expression. [R.2.27]

There are two common errors with this:

* + - 1. Multiplying error (INCORRECT!)
      2. Sign error (INCORRECT!)

NOTE: A negative exponent does NOT make a negative number!

A negative exponent means **reciprocal** (flip it!).

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| Write it as a fraction. | Negative exponent means **“take the stairs”** (reciprocal).  BASE (6) is the SAME;  EXPONENT (– 3) changes sign. | Simplify – “do it.” | **Answer** |

Note also that any time you are dealing with constants (no variables), you can verify the result using your calculator:

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* **EXAMPLE:** Use the quotient rule to simplify the expression. [R.2.33]

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| Negative exponent means **“take the stairs”** (reciprocal).  Both bases (6) and (7) will take the stairs – they will switch places. | Simplify – “do it.” | (reduce fraction, if necessary.)  **Answer** |

Note also that any time you are dealing with constants (no variables), you can verify the result using your calculator:

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* **EXAMPLE:** Use the product rule to simplify. [R.2.39]

(Type exponential notation with positive exponents.)

There are two common errors with this:

1. Multiplying bases error (INCORRECT!)
2. Multiplying exponents error (INCORRECT!)

* **Product Rule:**

When multiplying powers w/same base, **ADD** exponents.

**Answer**

NOTE: because the solution must be in *exponential notation*, using the calculator isn’t helpful.

* **EXAMPLE:** Multiply and simplify. [R.2.35]

(Simplify your answer. Type exponential notation with positive exponents.)

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| * **Product Rule:**     When multiplying powers w/same base, **ADD** exponents. | Can’t have negative exponents!  Write as fraction.  “Take the stairs” (reciprocal). | **Answer**  solution must be in *exponential notation*  Leave answer with exponent – don’t “do it.” |

* **EXAMPLE:** Use the product rule to simplify. [R.2.37]

(Type exponential notation with positive exponents.)

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| Rearrange factors to multiply constants separately from variables. | Simplify.   * **Product Rule:**     When multiplying powers w/same base, **ADD** exponents. | **Answer**  (No negative exponents.) |

* **EXAMPLE:** Use the quotient rule to simplify the expression. [R.2-23]

Use positive exponents to write the answer.

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| Same base (4).   * **Quotient Rule:**     When dividing powers w/same base, **SUBTRACT** exponents. | Simplify. | Can’t have negative exponents!  Write as fraction.  “Take the stairs” (reciprocal). | **Solution**  Leave answer with exponent – don’t “do it.” |

**EASIER WAY? – RESET!**

* **EXAMPLE:** Use the ~~quotient rule~~ to simplify the expression. [R.2-23]

Use positive exponents to write the answer.

Rather than use the quotient rule, focus on the **negative exponent** in the numerator:

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| Because of the negative exponent, “take the stairs” (reciprocal).  Connect existing denominator with new piece using multiplication. | * **Product Rule:**     When multiplying powers w/same base, **ADD** exponents. | **Answer**  Leave answer with exponent – don’t “do it.” |

## Another way to do Division: “**Face-Off!”**

* **EXAMPLE:** Simplify the expression. Write the answer with only positive exponents.

All variables are nonzero. [R.2.69]

To simplify this expression, work in “zones” – constants (coefficients), variable *a*, variable *b*.

Then, merge (multiply) all the results together.

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| **Constants (coefficients).** | **Variable *a*.** | **Variable *b*.** | | **Merge.** |
|  |  | 1. | 2. | Merge together the results from constants (coefficients) and variables. |
| Simplify the fraction of coefficients, if possible. | **“Face-off!”**   1. Do both have positive exponents?     YES   1. Who has more, top or bottom? TOP 2. By how much? 2 3. Simplifies to   on TOP. | **“Face-off!”**   1. Do both have positive exponents? NO   “take the stairs”  (reciprocal)   1. Who has more, top or bottom? TOP 2. By how much? 3 3. Simplifies to   on TOP. | | constants  (coefficients):  Variable *a*:  Variable *b*:  **Merged**  or  **Answer** |

* **EXAMPLE:** Use the rules of exponents to simplify the expression. [R.2.77] (Type exponential notation with positive exponents.)

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| Try to simplify INSIDE parentheses first.   * Fraction is already simplified. * is fine … already has positive exponent | * needs fixing … because of the negative exponent, “take the stairs”   (reciprocal). | * Coefficients always have exponent of understood 1. * **Quotient to Power Rule**   The exponent (2) applies to numerator and denominator. | The POSITIVE exponent, 2, applies to ALL factors.   * **Power Rule:**     When raising a power to a power, **MULTIPLY** exponents.  Now simplify: | **Answer**  Carefully merge together all the separate calculations.  Be careful who goes in the numerator (TOP) and who goes in the denominator (BOTTOM). |

* **EXAMPLE:** Simplify and write with positive exponents. [R.2.73]

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| * Coefficients always have exponent of understood 1. * **Product to Power Rule:**     The exponent (– 3) applies to each factor in the parentheses. | * **Power Rule:**   When raising a power to a power, **MULTIPLY** exponents.  Now, write as fraction. | and need fixing … because of the negative exponent, “take the stairs” (reciprocal). | Finally, just need to simplify  in denominator.    **Answer** |

Sources Used:

1. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
2. Powers of 10 chart, <https://www.eduplace.com/math/mw/background/6/01/te_6_01_overview.html>